

The possible candidates of tetraquark : $Z_b(10610)$ and $Z_b(10650)$

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Using the chromomagnetic interaction Hamiltonian with proper account for the $SU(3)$ flavor symmetry breaking, we have performed a schematic study on the masses of S -wave heavy tetraquarks as $bq\bar{b}\bar{q}$ (q denotes u, d, s quark). It is found that the numeral results for $bub\bar{d}$ or $bdb\bar{u}$ with 1^+ quantum number are 10612 MeV and 10683 MeV respectively, which are well compatible with the recent detected charged bottomonium-like $Z_b(10610)$ and $Z_b(10650)$. Theoretically, we also investigate the possible tetraquark states of 1^{++} and 2^+ due to the charge conjugation as the potential candidates for the updating experiments.

I. INTRODUCTION

In the recent discovery from the Belle Collaboration, two charged bottomonium-like resonances have been announced, dubbed $Z_b(10610)$ and $Z_b(10650)$, in the $\pi^\pm\Upsilon(nS)$ ($n = 1, 2, 3$) and $\pi^\pm h_b(mP)$ ($m = 1, 2$) mass spectra [1]. In detail, the weighted average masses and widths are $M[Z_b(10610)] = 10608.4 \pm 2.0\text{MeV}$, $\Gamma[Z_b(10610)] = 15.6 \pm 2.5\text{MeV}$ and $M[Z_b(10650)] = 10653.2 \pm 1.5\text{MeV}$, $\Gamma[Z_b(10650)] = 14.4 \pm 3.2\text{MeV}$ respectively. Exclude the hypotheses of $J \leq 2$ via the comparison of angular distributions, the quantum numbers for both $Z_b(10610)$ and $Z_b(10650)$ favored 1^+ [1]. The fact that the non-zero electric charged states cannot be explained as conventional quarkonium systems, has been encouraged great interest as exotic states. Differ with the previous theoretical works [1–5] exploring the two states as a meson-meson molecules as well as the mixture of molecules with bottomonium [6], we provide a trivial research within the tetraquark framework regarding the two charged states as the possible candidates of tetraquark.

The possible existence of tetraquark states has recently received a renewed interest due to the new resonances in the spectrum of states with open and hidden charm. The picture of tetraquark was foremost raised and was used to describe scalar mesons below 1GeV in 1977 by Jaffe[7–9]. In the special models, it is suggested that the $X(3872)$ can be a tetraquark state of $cq\bar{c}\bar{q}$, where q denotes a light quark, with 1^{++} quantum numbers[10–13]. Besides $X(3940)$ is predicted as a possible tetraquark state of 2^{++} [10], explicitly, $Y(4140)$ can be interpreted as a ground state of $c\bar{s}\bar{c}s$ [14] as well as $Y(4260)$ particle can be the first orbital excitation of a diquark-antidiquark state $c\bar{s}\bar{c}s$ [15]. Furthermore, another interesting the $Z(4430)$ state, which has positive or negative charge, has been regarded as the first radial excitation of $cu\bar{c}\bar{d}$ or $cu\bar{c}\bar{d}$ [16–18] etc. If these states can be confirmed as tetraquarks,

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similar with the charmonium-like $X(3872)$ assuming $cq\bar{c}\bar{q}$ structure, tetraquark framework also holds with the possibility of $bq\bar{b}\bar{q}$ to the bottomonium-like states.

In this work, we perform a detailed study of the mass splitting of the $bq\bar{b}\bar{q}$ tetraquarks for $J^P = 1^+$ and $J^P = 2^+$ quantum numbers in the $SU(3) \otimes SU(2)$ color-spin representations. In the following part, we briefly review the simple quark model involved in the chromomagnetic interaction Hamiltonian matrix. Then, the mass spectrum of $bq\bar{b}\bar{q}$ tetraquarks are calculated with account for flavor-symmetry breaking. Some discussions are summarized in Sec.IV, as a brief conclusion.

II. THE TETRAQUARK MASS SPECTRUM OF TYPE $bq\bar{b}\bar{q}$

The chromomagnetic interaction arising from one-gluon exchange in MIT bag model is quite successful in the description the mass splitting of meson and baryon spectrum [19]. The interaction Hamiltonian acting on the colour and spin degrees of freedom reads [8, 19]:

$$H_{cm} = - \sum_{i>j} v_{ij} \tilde{\lambda}_i^c \cdot \tilde{\lambda}_j^c \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (1)$$

where $\vec{\sigma}_i$ is the quark spin operator and $\tilde{\lambda}_i^c$ is the color operator for the i -th quark. The values of the coefficients v_{ij} depend on the quark masses and properties of the special wave function of the multiquark system. To be precise, if i or j indicates an antiquark, the following replacement should be understood:

$$\tilde{\lambda}_i \rightarrow -\tilde{\lambda}_i^*, \quad \vec{\sigma}_i \rightarrow -\vec{\sigma}_i^*. \quad (2)$$

Accordingly, the mass of tetraquark is given by the expectation value of the colourmagnetic model Hamiltonian [11]:

$$H = \sum_i m_i + H_{cm}, \quad (3)$$

where m_i is effective mass of the i -th constituent quark. Subsequently, the m_i in Eq.(3) and v_{ij} in Eq.(1) as parameters can be extracted from heavy mesons in the following calculation.

Here we simply use chromomagnetic interaction to fit heavy mesons experimental masses from which the constituent effective mass m_i and the coefficient v_{ij} can be obtained for the $bq\bar{b}\bar{q}$ tetraquark system. Using the corresponding experimental values of J/Ψ , η_c , Υ , η_b , B^* , B^\pm , B_s^* , B_s^0 , and B_c^\pm masses [20]. We use Eq.(3) to extract these parameters:

$$\begin{aligned} m_{J/\Psi} &= 2m_c + \frac{16}{3}v_{c\bar{c}}, & m_{\eta_c} &= 2m_c - 16v_{c\bar{c}}, \\ m_\Upsilon &= 2m_b + \frac{16}{3}v_{b\bar{b}}, & m_{\eta_b} &= 2m_b - 16v_{b\bar{b}}, \\ m_{B^*} &= m_u + m_b + \frac{16}{3}v_{b\bar{u}}, & m_{B^\pm} &= m_u + m_b - 16v_{b\bar{u}}, \\ m_{B_s^*} &= m_s + m_b + \frac{16}{3}v_{b\bar{s}}, & m_{B_s^0} &= m_s + m_b - 16v_{b\bar{s}}, \\ m_{B_c^\pm} &= m_c + m_b - 16v_{b\bar{c}}. \end{aligned} \quad (4)$$

Therefore, we have

$$\begin{aligned} m_u &= 592.14\text{MeV}, & m_s &= 681.65\text{MeV}, & m_c &= 1533.88\text{MeV}, & m_b &= 4721.47\text{MeV}, \\ v_{c\bar{c}} &= 5.47\text{MeV}, & v_{b\bar{u}} &= 2.15\text{MeV}, & v_{b\bar{s}} &= 2.30\text{MeV}, & v_{b\bar{c}} &= 4.06\text{MeV}, \\ v_{b\bar{b}} &= 3.25\text{MeV}, & & & & & & \end{aligned} \quad (5)$$

where $v_{c\bar{c}}$, $v_{b\bar{u}}$, $v_{b\bar{s}}$, $v_{b\bar{c}}$, and $v_{b\bar{b}}$ are the coefficient in Eq.(1) for $c\bar{c}$, $b\bar{u}$, $b\bar{s}$, $b\bar{c}$, and $b\bar{b}$ meson system respectively. On the other hand, we can easily obtain $v_{q\bar{q}} = 29.8\text{MeV}$ from fitting the experimental values of light mesons.

Below we define the total color-spin wave function of tetraquark and give the colormagnetic Hamiltonian matrix. There are a set of $J^P = 0^+$, 1^+ and 2^+ quantum numbers for the possible ground state tetraquark. In the case of color-singlet, all conceivable color-spin states have been constructed for given these quantum numbers [14]. Specially, we use the same notation which has been introduced in Ref. [11] for the basis vectors and only write $J^P = 1^+$ and 2^+ states in the work. For $J^P = 1^+$, the basis can be built with $(1, 3)$ and $(2, 4)$ subsystems:

$$\begin{aligned} \alpha_1 &= [(q_1\bar{q}_3)_0^1 \otimes (q_2\bar{q}_4)_1^1]_1^1, & \alpha_2 &= [(q_1\bar{q}_3)_1^1 \otimes (q_2\bar{q}_4)_0^1]_1^1, \\ \alpha_3 &= [(q_1\bar{q}_3)_1^1 \otimes (q_2\bar{q}_4)_1^1]_1^1, & \alpha_4 &= [(q_1\bar{q}_3)_0^8 \otimes (q_2\bar{q}_4)_1^8]_1^1, \\ \alpha_5 &= [(q_1\bar{q}_3)_1^8 \otimes (q_2\bar{q}_4)_0^8]_1^1, & \alpha_6 &= [(q_1\bar{q}_3)_1^8 \otimes (q_2\bar{q}_4)_1^8]_1^1, \end{aligned} \quad (6)$$

where the superscript and subscript indicate a well defined color and spin, respectively. For $bq\bar{b}q$ (q and \bar{q} denote quark and anti-quark of the same flavor) states, where α_3 and α_6 have charge conjugation $C = +1$ and $\alpha_1, \alpha_2, \alpha_4, \alpha_5$ have conjugation $C = -1$ as explained in Ref. [13, 14]. Similarly, the state of $J^P = 1^+$ can be also rewritten in the $(1, 4)$ and $(2, 3)$ basis, not given detailed form here, which can be easily gained from Ref. [11]. The chromomagnetic interaction Hamiltonian H_{cm} acting on this basis vectors (6) can be written the following blocks matrix

$$H_{cm} = - \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad (7)$$

with 3×3 submatrices

$$A_{11} = \begin{pmatrix} \frac{16}{3}(3v_{13} - v_{24}) & 0 & 0 \\ 0 & -\frac{16}{3}(v_{13} - 3v_{24}) & 0 \\ 0 & 0 & -\frac{16}{3}(v_{13} + 3v_{24}) \end{pmatrix}, \quad (8)$$

$$A_{12} = A_{21}^\dagger = \begin{pmatrix} 0 & \frac{4\sqrt{2}}{3}(v_{12} + v_{34} + v_{14} + v_{23}) & -\frac{8}{3}(v_{12} - v_{34} - v_{14} + v_{23}) \\ \frac{4\sqrt{2}}{3}(v_{12} + v_{34} + v_{14} + v_{23}) & 0 & -\frac{8}{3}(v_{12} - v_{34} + v_{14} - v_{23}) \\ -\frac{8}{3}(v_{12} - v_{34} - v_{14} + v_{23}) & -\frac{8}{3}(v_{12} - v_{34} + v_{14} - v_{23}) & -\frac{4\sqrt{2}}{3}(v_{12} + v_{34} - v_{14} - v_{23}) \end{pmatrix}, \quad (9)$$

$$A_{22} = \begin{pmatrix} -2v_{13} + \frac{2}{3}v_{24} & -\frac{4}{3}(v_{12} + v_{34}) + \frac{14}{3}(v_{14} + v_{23}) & \frac{4\sqrt{2}}{3}(v_{12} - v_{34}) + \frac{14\sqrt{2}}{3}(v_{14} - v_{23}) \\ -\frac{4}{3}(v_{12} + v_{34}) + \frac{14}{3}(v_{14} + v_{23}) & \frac{2}{3}v_{13} - 2v_{24} & -\frac{4\sqrt{2}}{3}(v_{12} - v_{34}) + \frac{14\sqrt{2}}{3}(v_{14} - v_{23}) \\ \frac{4\sqrt{2}}{3}(v_{12} - v_{34}) + \frac{14\sqrt{2}}{3}(v_{14} - v_{23}) & -\frac{4\sqrt{2}}{3}(v_{12} - v_{34}) + \frac{14\sqrt{2}}{3}(v_{14} - v_{23}) & \frac{2}{3}(v_{13} + v_{24}) + \frac{4}{3}(v_{12} + v_{34}) + \frac{14}{3}(v_{14} + v_{23}) \end{pmatrix}. \quad (10)$$

For $J^P = 2^+$ the situation is more simple. There are two linearly independent basis vectors:

$$\beta_1 = [(q_1\bar{q}_3)_1^1 \otimes (q_2\bar{q}_4)_1^1]_2^1, \quad \beta_2 = [(q_1\bar{q}_3)_1^8 \otimes (q_2\bar{q}_4)_1^8]_{12}^1. \quad (11)$$

The corresponding H_{cm} acting on this basis (11) can be written the 2×2 matrix

$$H_{cm} = - \begin{pmatrix} -\frac{16}{3}(v_{13} + v_{24}) & -\frac{4}{3}\sqrt{2}(v_{12} + v_{34} - v_{14} - v_{23}) \\ -\frac{4}{3}\sqrt{2}(v_{12} + v_{34} - v_{14} - v_{23}) & \frac{2}{3}(v_{13} + v_{24}) - \frac{4}{3}(v_{12} + v_{34}) - \frac{14}{3}(v_{14} + v_{23}) \end{pmatrix}. \quad (12)$$

As such, the $(1, 4)$ and $(2, 3)$ basis are not given detailed form for the tetraquark states of $J^P = 2^+$. If the reader is interested in the question, please refer to the appendix part in Ref. [14] where there is accurate identification.

III. NUMERICAL RESULTS AND DISCUSSION

Here we respectively diagonalize the above 6×6 and 2×2 Hamiltonian matrix elements in the complete tetraquark configuration space of type $bq\bar{b}\bar{q}$ with the 1^+ and 2^+ quantum numbers. In the calculation of the matrix elements with account for the heavy quark limit, we approximately consider that the parameters $v_{b\bar{q}}$ and v_{bq} are equal because their difference is very small in the heavy quark limit. Therefore, we can obtain the chromomagnetic interaction eigenvalues (E) and the masses spectrum (M) of tetraquarks of type $bq\bar{b}\bar{q}$ for 1^+ quantum numbers which have been exhibited in Table I. Furthermore, we can observe there are masses of several states in the range $10550 \sim 10700$ MeV. Concretely, we will discuss these theoretical tetraquark states as the possible charged bottomonium-like resonances, both $Z_b(10610)$ and $Z_b(10650)$, as follows.

TABLE I: Chromomagnetic interaction eigenvalues (E), the masses (M) and amplitudes of the basis vectors (6) of $bq\bar{b}\bar{q}$ with 1^+ quantum numbers

J^P	$E(\text{MeV})$	$M(\text{MeV})$	α_1	α_2	α_3	α_4	α_5	α_6
1^+	-460.056	10167.9	-0.00003	-0.99934	0	-0.03631	-0.00101	0
1^+	-15.670	10612.3	-0.02616	0.03578	0	-0.97924	-0.19780	0
1^+	55.348	10683.3	0.29403	0.00619	0	-0.19657	0.93534	0
1^+	111.911	10739.9	-0.95544	0.00096	0	-0.03368	0.29327	0
1^+	-47.833	10580.2	0	0	0	0	0	1
1^+	176.267	10804.3	0	0	1	0	0	0

For the 1^+ states, as it can be seen from Table I, there are three states at 10580 MeV, 10612 MeV and 10683 MeV which are quite close to the experimental values of two narrow charged structures $Z_b(10610)$ and $Z_b(10650)$. Among the three states, the lowest state at 10580 MeV has an appropriate mass for the experimental range of $Z_b(10610)$, but the wave function of this state only is α_6 , a pure octet-octet hidden color state, which obviously cannot dissociate into a bottomonium state and a light pseudoscalar meson. Thus the $Z_b(10610)$ is disfavored for the 1^+ tetraquark state at 10580 MeV. For both states of 10612 MeV and 10683 MeV, which are also commendably close to the experimental range, their structures can be composed of $bub\bar{d}$ or $bdb\bar{u}$. As seen from Table I, both states have a very small component on the state α_2 , and therefore they have a narrow width in $\Upsilon\pi$ decay channel

as well as they can be taken as the best candidates for $Z_b(10610)$ and $Z_b(10650)$ from Belle Collaboration[1].

It is worthwhile to notice that the above three states are not the lowest-lying state for $bq\bar{b}\bar{q}$ system. The lowest state at 10167.9 MeV is strongly coupled to the channel consisting of a bottomonium state and a light pseudoscalar meson because there is very big amplitude of the basis vector α_2 . Therefore, this state is probably very broad and fall parts very easily. Thus it is not easy to observe it experimentally.

In the case of q and \bar{q} being the same flavor, the 1^+ state can be separated 1^{++} and 1^{+-} states from the general consideration of charge conjugation symmetry. The lowest 1^{++} state at 10580 MeV is entirely analogous to $X(3872)$ as $cq\bar{c}q$ tetraquark state. In Table I, $V_{bq} = V_{b\bar{q}}$ are assumed, this states only is α_6 , a hidden color state, but if $\Delta = V_{bq} - V_{b\bar{q}}$ have a nonvanishing, but small, value, the state mass is nonsensitive to the change of Δ from Table II. Meanwhile, this state shall receive the small component coming from α_3 , as shown in Table II, which implies that this state can decay into $\Upsilon + \rho$ or $\Upsilon + \omega$ with the narrow width.

Such as we can predict there is a state of tetraquark of 2^+ at 10631 MeV analogous to the $X(3940)$ [10]. As it can be seen from Table III, the lowest state is a almost pure hidden color state in the $b\bar{b} + q\bar{q}$ channel even if $v_{b\bar{q}}$ and v_{bq} are not equal.

TABLE II: Chromomagnetic interaction eigenvalues (E), the masses (M) and amplitudes of the basis vectors (6) of the lowest 1^{++} $bq\bar{b}\bar{q}$ state with the different Δ .

$\Delta(\text{MeV})$	$E(\text{MeV})$	$M(\text{MeV})$	α_1	α_2	α_3	α_4	α_5	α_6
-1.05	-45.104	10582.9	0	0	0.01788	0	0	0.99984
1.05	-50.702	10577.3	0	0	-0.01744	0	0	0.99985

TABLE III: Chromomagnetic interaction eigenvalues (E), the masses (M) and amplitudes of the basis vectors (6) of $bq\bar{b}\bar{q}$ with 2^+ quantum numbers

J^P	$E(\text{MeV})$	$M(\text{MeV})$	β_1	β_2
2^+	3.767	10631.0	0	1
2^+	176.267	10803.5	1	0

IV. SUMMARY

In this work, the mass spectrum of tetraquark of type $bq\bar{b}\bar{q}$ with 1^+ and 2^+ quantum numbers have been calculated by using the simple chromomagnetic interaction with proper account for the $SU(3)$ flavor symmetry breaking. The numerical result both 10612 MeV and 10683 MeV are well compatible with the experimental values for describing the charged bottomonium-like $Z_b(10610)$ and $Z_b(10650)$, which should have a minimum tetraquark content $b\bar{u}b\bar{d}$ or $b\bar{d}b\bar{u}$, respectively. At the same time, we also predict two possible tetraquark states of 1^{++} and 2^+ states, which have masses of 10580 MeV and 10631 MeV, and expect to search them in future experiments.

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